Statistics

Lecture 10

Feb 19-8:47 AM

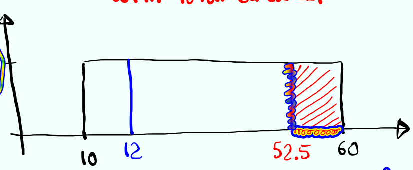
SG 17

Consider a uniform Prob. dist. For all values from 10 to 60.

$10 \le x \le 60$

Graph is rectangular with total area 1.

$\frac{1}{60-10} = \frac{1}{50}$



1) Find $P(x=12) = \boxed{0}$

Line

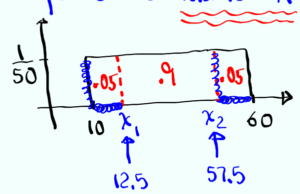
2) $P(x>52.5) = (60-52.5)\cdot\frac{1}{50}$

$= \frac{7.5}{50} = \boxed{.15} = \boxed{\frac{3}{20}}$

3) Find two x-values that separate the middle 90% from the rest.

$1 - .9 = .1$

$.1 \div 2 = .05$



$(x_1 - 10)\cdot\frac{1}{50} = .05$

$x_1 - 10 = 50(.05)$

$x_1 = 10 + 2.5$

$\boxed{x_1 = 12.5}$

$(60 - x_2)\cdot\frac{1}{50} = .05$

$60 - x_2 = 50(.05)$

$60 - 2.5 = x_2$

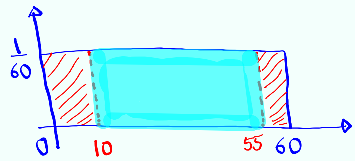
$\boxed{x_2 = 57.5}$

Oct 31-8:05 AM

wait time at DMV has a uniform prob. dist. and does not exceed 60 minutes

Rectangular

$0 \le x \le 60$



P(wait time is less than 10 minutes or more than 55 minutes).

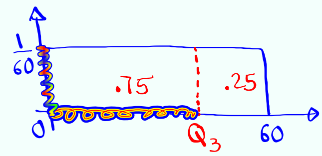
$P(x<10 \text{ OR } x>55) = 1 - P(10<x<55)$

$= 1 - (55-10)\cdot\frac{1}{60} = 1 - \frac{45}{60}$

$= 1 - \frac{3}{4} = \boxed{\frac{1}{4}} = \boxed{.25}$

find $x = Q_3$

75% below
25% above



$(Q_3 - 0)\cdot\frac{1}{60} = .75$

$Q_3 = 60(.75) \rightarrow \boxed{Q_3 = 45}$

Oct 31-8:16 AM

Standard Normal Prob. Dist.:

1) use Z, $P(Z=c)=0$

2) Graph is symmetric, bell-shape with total area $=1$.

3) Mean = Mode = Median

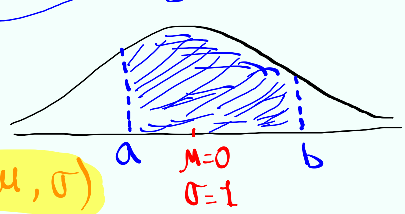
4) $\mu=0$, $\sigma=1$

$\boxed{P(a<Z<b)}$ is the corresponding area within the graph.

using TI:

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

normalcdf(L, U, μ, σ)

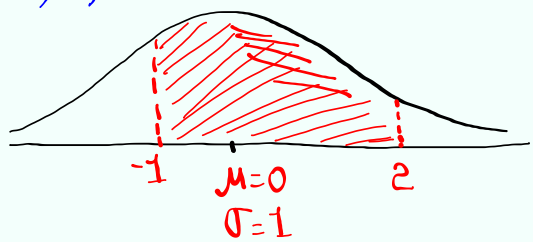


Oct 31-8:25 AM

find $P(-1 < Z < 2)$

$= \text{normalcdf}(-1, 2, 0, 1)$

(−)

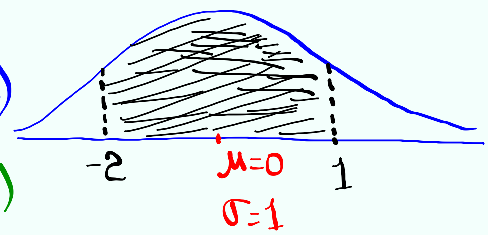


$= \boxed{.819}$

find $P(-2 < Z < 1)$

$= \text{normalcdf}(-2, 1, 0, 1)$

(−)



$= \boxed{.819}$

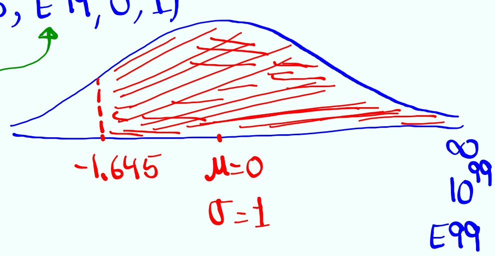
Oct 31-8:31 AM

find $P(Z > -1.645)$

$= \text{normalcdf}(-1.645, E99, 0, 1)$

(−)

$\boxed{2nd}\boxed{9}$



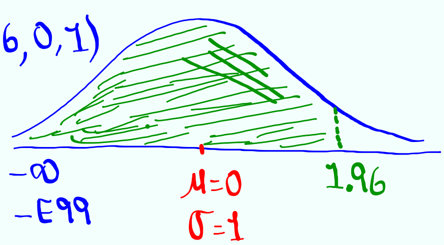
$= \boxed{.950}$

find $P(Z < 1.96)$

$= \text{normalcdf}(-E99, 1.96, 0, 1)$

(−)

$\boxed{2nd}\boxed{9}$



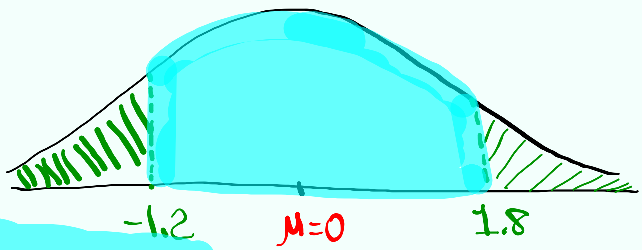
$= \boxed{.975}$

Oct 31-8:38 AM

$P(Z < -1.2 \text{ OR } Z > 1.8)$ → If it was AND $\boxed{0}$

$= 1 - P(-1.2 < Z < 1.8)$

Total Area
Total Prob.



$= 1 - \text{normalcdf}(-1.2, 1.8, 0, 1)$

$= \boxed{.151}$

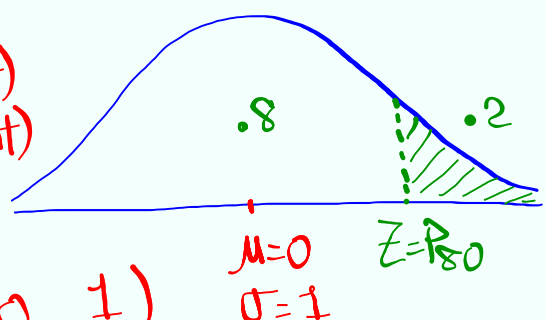
Oct 31-8:46 AM

Doing Reverse

find $Z = P_{80}$

80% below (Left)
20% above (Right)

2nd VARS



$Z = \text{invNorm}(.8, 0, 1) = \boxed{.842}$

left Area μ σ

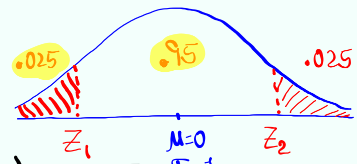
Oct 31-8:50 AM

Find two Z-Values that separate the middle 95% from the rest.

→ Round to 3-dec. Places

$1 - .95 = .05$

$.05 \div 2 = .025$



$Z_1 = \text{invNorm}(.025, 0, 1) = \boxed{-1.960}$

$Z_2 = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

If $-2 \leq Z \leq 2$ → usual data element

usual Range → 95% Range

$P(-1.96 < Z < 1.96) = .95$

SG 17 ✓

Oct 31-8:55 AM

SG 18

Normal Prob. dist.

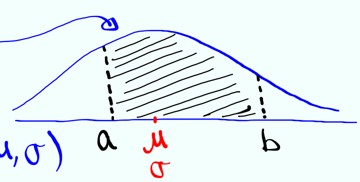
1) use x, $P(x=c)=0$

2) data dist. is symmetric, bell-Shape with total area $=1$.

3) Mean = Mode = Median

4) μ & σ are given in the problem

$\boxed{P(a<x<b)}$ is the Corresponding area within the normal curve.



Use TI

2nd VARS

$\text{normalcdf}(L, U, \mu, \sigma)$

$N(\mu, \sigma)$

Normal · Mean · Standard Dev.

Oct 31-9:16 AM

Given $N(82, 6)$

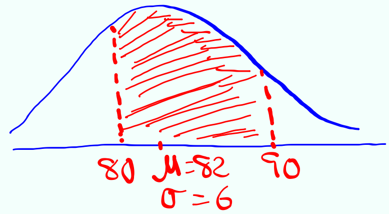
N = Normal Prob. dist.

$82 = \mu$, $6 = \sigma$

find $P(80 < x < 90)$

$= \text{normalcdf}(80, 90, 82, 6)$

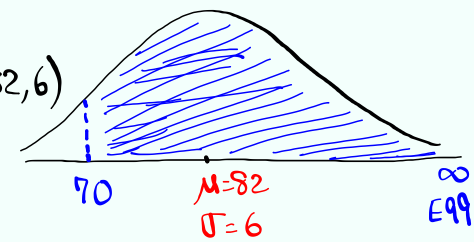
$= \boxed{.539}$



find $P(x > 70)$

$= \text{normalcdf}(70, E99, 82, 6)$

$= \boxed{.977}$



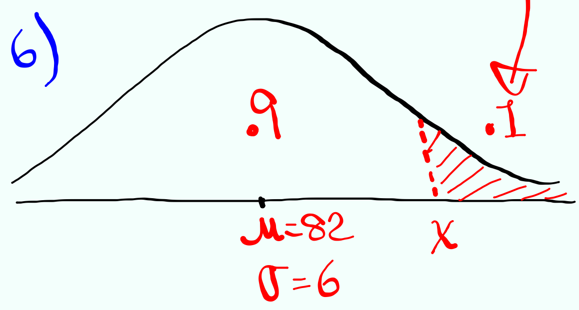
Oct 31-9:22 AM

find a value, rounded to whole #, that separates the top 10% from the rest.

$x = \text{invNorm}(.9, 82, 6)$

$= 89.689$

$\approx \boxed{90}$



Oct 31-9:27 AM

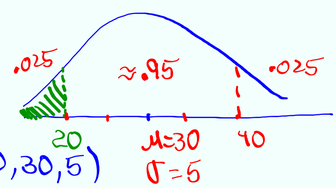
Ages of college students has a normal Prob. dist with mean of 30 years and Stand. dev. of 5 Yrs.

$N(30, 5)$

If we randomly Select one student, find the Prob. that his/her age is

a) below 20.

$P(x < 20)$

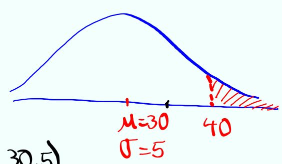


$= \text{normalcdf}(-E99, 20, 30, 5)$

$= \boxed{.023}$

b) exceeds 40

$P(x > 40)$



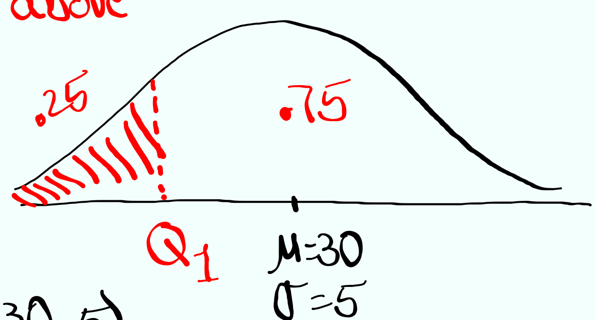
$= \text{normalcdf}(40, E99, 30, 5) = \boxed{.023}$

Oct 31-9:31 AM

Find Q_1, Round to whole #

25% below

75% above



$Q_1 = \text{invNorm}(.25, 30, 5)$

$\approx 26.628 \approx \boxed{27}$

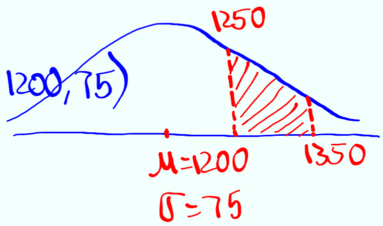
Oct 31-9:40 AM

SAT Scores are normally distributed with mean of 1200 and Standard dev. of 75. $N(1200, 75)$

If we randomly select one SAT exam, find the Prob. that Score is between 1250 and 1350.

$P(1250 < x < 1350)$

$= \text{normalcdf}(1250, 1350, 1200, 75)$



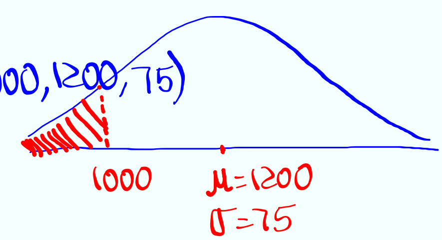
$= \boxed{.230}$

Oct 31-9:43 AM

P(below 1000)

$P(x < 1000)$

$= \text{normalcdf}(-E99, 1000, 1200, 75)$

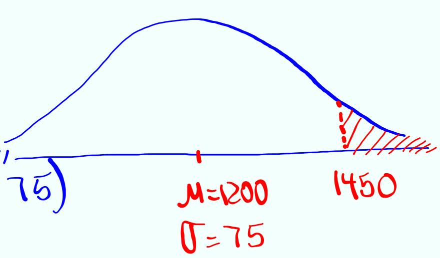


$= \boxed{.004}$

P(above 1450)

$P(x > 1450)$

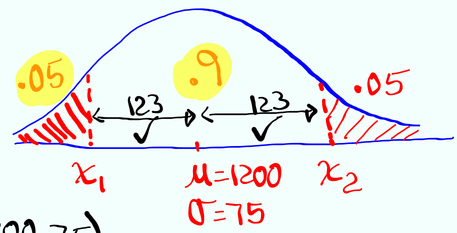
$= \text{normalcdf}(1450, E99, 1200, 75)$



$= \boxed{4.3 \times 10^{-4}}$

Oct 31-9:49 AM

Find two SAT Scores, Round to whole #, that Separate the middle 90% from the rest.



$x_1 = \text{invNorm}(.05, 1200, 75) = \boxed{1077}$

$x_2 = \text{invNorm}(.95, 1200, 75) = \boxed{1323} = 1200+123$

SG 18 ✓

Oct 31-9:55 AM

SG 19

Central Limit Theorem:

Clear all lists,

Store 2,4,6, and 8 in L1,

use $\boxed{\text{1-Var Stats}}$ with L1 only to find

$\mu = \bar{x} = \boxed{5}$ $\quad \sigma = \sigma_x = 2.236$ $\quad \sigma^2 = 5$

VARS

5: Statistics

4: σ_x $\quad x^2$

Enter

Take all Samples of Size 2 with replacement from 2,4,6, and 8.

2,2	2,4	2,6	2,8
4,2	4,4	4,6	4,8
6,2	6,4	6,6	6,8
8,2	8,4	8,6	8,8

16 Samples

Oct 31-10:16 AM

Now find $\bar{x}$ of each sample

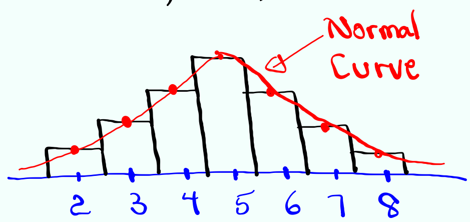
2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

$16\ \bar{x}$

$\bar{x}$	$P(\bar{x})$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16

Draw Prob. dist. hist.

$\bar{x} \to$ MP , $P(\bar{x}) \to$ Rel. F



$\bar{x} \to L2$, $P(\bar{x}) \to L3$

use $\boxed{\text{1-Var Stats}}$ with L2 & L3

$\mu_{\bar{x}} = 5$ $\quad \sigma_{\bar{x}} = 1.581$ $\quad \sigma^2_{\bar{x}} = 2.5 = \frac{5}{2}$

Oct 31-10:23 AM

Clear all lists.

Store 2, 4, 6, 8, and 10 in L1.

use $\boxed{\text{1-Var Stats}}$ with L1 only.

Find

$\mu = 6$ $\quad \sigma = 2.828$ $\quad \sigma^2 = 8$

take all samples of size 2 with replacement from 2, 4, 6, 8, and 10.

2,2	2,4	2,6	2,8	2,10
4,2	4,4	4,6	4,8	4,10
6,2	6,4	6,6	6,8	6,10
8,2	8,4	8,6	8,8	8,10
10,2	10,4	10,6	10,8	10,10

we have 25 samples of size 2.

Oct 31-10:33 AM

Find $\bar{x}$ of each Sample.

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

$\bar{x}$	$P(\bar{x})$
2	1/25
3	2/25
4	3/25
5	4/25
6	5/25
7	4/25
8	3/25
9	2/25
10	1/25

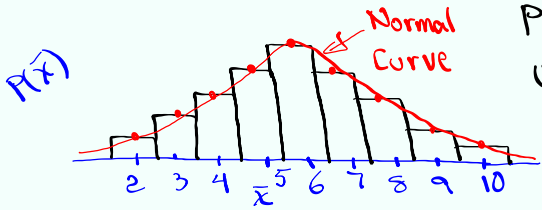
25 $\bar{x}$s.

Draw Prob. dist. hist.

$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Use L2 & L3 to find



$\mu_{\bar{x}} = 6$ $\quad \sigma_{\bar{x}} = 2$ $\quad \sigma^2_{\bar{x}} = 4 = \frac{8}{2}$

Oct 31-10:39 AM

Central Limit Theorem

$$\boxed{\mu_{\bar{x}} = \mu}$$

$$\boxed{\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}}$$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

Suppose we have a normal dist. with $\mu = 85$ and $\sigma = 10$.

We take all Samples of Size 4

$\mu_{\bar{x}} = \mu = \boxed{85}$ $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{4}} = \frac{10}{2} = \boxed{5}$

Oct 31-10:49 AM

Ages of nurses are normally dist. with mean of 40 and standard dev. of 8.

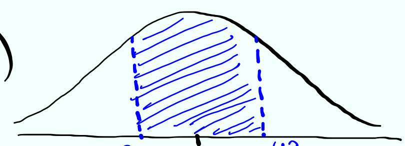
$N(40, 8)$

If we take samples of size 4, find the prob. that their mean age is between 38 and 42.

$P(38 < \bar{x} < 42)$

$= \text{normalcdf}(38, 42, 40, 4)$

$= \boxed{.383}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 40 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8}{\sqrt{4}} = 4 \end{cases}$

Oct 31-10:54 AM

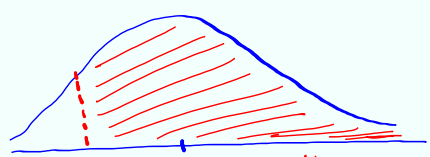
Exam scores are normally dist. with $\mu = 84$ & $\sigma = 10$.

If we randomly select 2 exams, find the prob. that their mean score is more than 78.

$P(\bar{x} > 78)$

$= \text{normalcdf}(78, E99, 84, 10/\sqrt{2})$

$= \boxed{.802}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 84 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{2}} \end{cases}$

SG 19 ✓

Oct 31-11:01 AM